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Mathematical Modeling of Groundwater Pollution

With 104 Illustrations

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1.1 Groundwater Quality

The chemical and biological constituents contained in groundwater depend on two factors: the natural environment of groundwater storage and movement, and human activities. Precipitation infiltration and surface water percolation are the natural sources of groundwater. The total dissolved solids (TDS) of precipitation is generally very low, but its chemical components will be changed when infiltrated through soil beds by a series of actions, such as adsorption, oxidation, reduction, ion exchange, and so on. The infiltration and exfiltration water will be involved in groundwater movement in both the vertical and the lateral directions in the aquifer. During this process, the TDS of groundwater will continually increase as rocks and minerals are dissolved in the water. Human activities may change the natural process and cause groundwater to contain organisms, hydrocarbons, heavy metals and other solid matter. Groundwater, therefore, should be looked upon as a multi-component fluid. The content of each component in groundwater can be assessed by its concentration, i.e., a mass of certain component contained in a volume of water (M/L^3). If the concentration of component α is written as C_α , then the standard of water quality for a certain use can be written in the following common form:

$$C_{\alpha, \min} \leq C_\alpha \leq C_{\alpha, \max} \quad (1.1.1)$$

($\alpha = 1, 2, \dots, n$)

$C_{\alpha, \min}$ and $C_{\alpha, \max}$ are the given lower and upper limits, respectively, of concentration of component α , and n is the total number of components considered. According to the actual situation, the components may refer to single ions or multi-ion compositions.

Groundwater quality (including its physical, chemical, and biological characteristics) has been changed so that it is no longer suited to the previous standard. The groundwater is said to be polluted.

Groundwater is buried under the land surface. It is not in direct contact with the surface water. Because of its low flow rate, the pollutants